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ERRORS DUE TO COUNTING STATISTICS IN THE TRIAXIAL STRAIN (STRESS) TENSOR DETERMINED BY DIFFRACTION

BY

P. Rudnik and J. B. Cohen

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ERRORS DUE TO COUNTING STATISTICS IN THE TRIAXIAL STRAIN (STRESS) TENSOR DETERMINED BY DIFFRACTION

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INTRODUCTION

Knowledge of the errors in a diffraction measurement of residual strains and stresses is useful information, not only in its own right, but also because it permits automation of a measurement to an operator specified precision. There are three sources of these errors:

- (1) Instrumental effects; primarily due to sample displacement, separation of the 0 and 20 axes of the diffractometer, and beam divergence. All three can be estimated², or minimized by employing parallel beam geometry.³
- (2) Uncertainties in x-ray elastic constants; which can now be evaluated.
- and (3) Errors in the diffraction peak position related to counting statistics. Equations to evaluate this source have been developed in Ref. 1 for the case of a stress state for which all σ_{i3} (i = 1,2,3) = 0, with the direction "3" normal to the sample surface, see Fig. 1. This means that the stresses lie only in the surface, e.g., a biaxial stress state $|\sigma_{i1}| |\sigma_{i2}| |\sigma_{i3}|$. There are, however, $|\sigma_{i2}| |\sigma_{i3}| |\sigma_{i4}| |\sigma_{i4}|$

numerous situations when the normal components are appreciable in an x-ray measurement^{5,6} and this is generally the case for neutron diffraction because with neutrons the beam can sample a sizeable volume, at a significant depth below the surface⁷. It

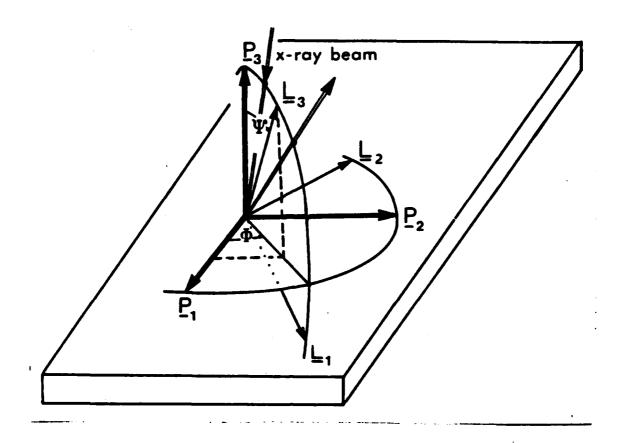


FIG. 1: The axial system. Strains are measured with diffraction by measuring the change in spacing of planes normal to the L₂ direction. (The axes P₁ define the specimen surface.)

TABI	E I: STRES	S TENSORS FOR SPECI	-		VIAT:	ions)
		DATA S	ET 1			
537.62	(161.94)	-24.03	(78.81)	-39	.15	(4.58)
		550.04	(161.66)	:	2.31	(3.56)
				78.	.29	(130.57)
		DATA S	ET 2			-
520.60	(137.25)	-4.03	(66.60)	-34	.17	(3.21)
		555.19	(137.03)	0.	.11	(2.69)
					.20	(110.67)
		DATA S	ET 3			•====
535.03	(158.28)	-20.13	(77.06)	-40	.19	(5.72)
		555.98	(157.99)	-0.	.98	(4.56)
			• •		.66	
		DATA S	ET 4	-		•
538.53	(146.23)		(70.95)	-38	.03	(3.83)
	•		(146.14)			(3.89)
			• • • • • • • • • • • • • • • • • • • •		.18	
AVERAC	SE .			REFERENCE	5	
	·	37.89		541	-20	-38
-		0.55			565	1
		83.83				86

^{*} values given in MPa; $V(d_a)^{\frac{1}{2}} = 0.00016 \text{ A}$

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is the purpose of this paper to derive equations to evaluate the counting statistical error for the entire three dimensional strain (or stress) tensor,

BASIC EQUATIONS

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We begin with the general equation for the strains (e_{ij}) and how these affect the interplanar spacing "d". (Refer to Fig. 1 for the axial system.) The measurement is made in the θ direction, with a sample tilted ϕ from the normal position (which is with the surface normal bisecting incident and scattered beams). Primed quantities refer to strains in the L_i co-ordinate system, unprimed terms are in the P_i system.

$$\langle \mathbf{e}_{33}^{\prime} \rangle_{\emptyset \psi} = (\mathbf{d}_{\emptyset \psi} - \mathbf{d}_{0})/\mathbf{d}_{0} = [\langle \mathbf{e}_{11} \rangle \cos^{2} \theta + \langle \mathbf{e}_{22} \rangle \sin^{2} \theta + \langle \mathbf{e}_{12} \rangle \sin^{2} \theta$$

$$-\langle \mathbf{e}_{23} \rangle] \sin^{2} \psi + \langle \mathbf{e}_{23} \rangle + [\langle \mathbf{e}_{13} \rangle \cos \theta + \langle \mathbf{e}_{23} \rangle \sin \theta] \sin^{2} \psi$$
 (1)

Note that the stress-free spacing, d_0 , is involved. While this term can be eliminated for a biaxial stress state, this is not possible for a general strain or stress tensor, and the reader may consult Ref. 8 for a discussion of problems associated with the measurement of this quantity. When c_1 , or c_1 , $\frac{1}{7}$, 0, c_2 , is not linear with $\sin^2\phi$ and has different curvature for $+\phi$ and $-\phi$. The carats imply that the strain values are averaged over the depth of penetration of the incident x-ray (neutron) beam and this is to be understood in what follows, as this additional notation is eliminated below.

Next, we define terms which involve measurements of $\mathbf{d}_{\phi, \phi}$ at plus and minus ϕ tilts of the surface normal. 5

$$a_{1} \equiv 1/2[e_{0\psi+}' + e_{0\psi-}'] = \{d_{0\psi+} + d_{0\psi-}')/2d_{0}\}-1$$

$$= e_{3,3} + [e_{1,1} \cos^{2}\theta + e_{2,2} \sin^{2}\theta + e_{1,2} \sin^{2}\theta - e_{3,3}])\sin^{2}\psi \qquad (2a)$$

Clearly, a_1 , should be linear with $\sin^2\phi$ and $c_{3,3}$ is the intercept, regardless of Φ .

$$a_{2} \equiv 1/2[\epsilon_{0\phi+} - \epsilon_{0\phi-}] = (d_{0\phi+} - d_{0,\phi-})/2d_{0}$$

$$= [\epsilon_{1,2} \cos\theta + \epsilon_{2,2} \sin\theta]\sin|2\phi|. \tag{2b}$$

Therefore, a is linear vs. sin 24.

Let:

$$\ell_1 = da_1/dsin^2\phi, \tag{3a}$$

$$\ell_2 = d_2/d\sin|2\phi| \tag{3b}$$

Then, at:
$$\phi = 0^{\circ}$$
, $_{0} \ell_{1} = \varepsilon_{11} - \varepsilon_{33}$,
 $\phi = 90^{\circ}$, $_{90} \ell_{1} = \varepsilon_{22} - \varepsilon_{33}$,
 $\phi = 45^{\circ}$, $_{43} \ell_{1} = 1/2(\varepsilon_{11} + \varepsilon_{22}) + \varepsilon_{12} - \varepsilon_{33}$,
 $= \varepsilon_{12} + 1/2(_{0} \ell_{1} + _{90} \ell_{1})$. (3c)

and similarly:

at
$$\Phi = 0^{\circ}$$
: $_{0} \ell_{2} = \varepsilon_{13}$,
at $\Phi = 90^{\circ}$: $_{20} \ell_{2} = \varepsilon_{23}$. (3d) \

Knowledge of the strain tensor permits the calculation of the stress components (σ_{ij}) from:

$$\sigma_{ij} = [1/2S_{2}(hk\ell)]^{-1} \{\epsilon_{ij} - \delta_{ij} \{S_{i}(hk\ell)/[3S_{i}(hk\ell) + 1/2S_{2}(hk\ell)]\} \cdot [\epsilon_{i1} + \epsilon_{i2} + \epsilon_{i3}]\}.$$
(4)

Here, δ_{ij} is the Kronecker delta function and S_i and $1/2S_2$ are the x-ray elastic constants which depend on the indices of the diffraction peak, hkl. (For an isotropic solid these values are -v/E and (1 + v)/E respectively.)

VARIANCES DUE TO COUNTING STATISTICS

For a function $X = f(x_1, x_2, x_3, ...)$, assuming the x_n are independent, the variance (V) is $\frac{9}{1}$:

$$V(X) = (\frac{dX}{dx_1})^2 V(x_1) + (\frac{dX}{dx_2})^2 V(x_2) + (\frac{dX}{dx_3})^2 V(x_3) + \dots$$
 (5)

For the straight line, y = mx + b, the slope and intercept is given by:

$$m = \frac{\frac{x}{1}(x_1 - \bar{x})(y_1 - \bar{y})}{\frac{x}{1}(x_1 - \bar{x})^a}$$
 (6a)

$$b = (2y_i - m2x_i)/N \tag{6b}$$

where N is the number of data points.

Employing Eq. (5):

$$V(m) = \begin{bmatrix} \frac{\Sigma}{\Sigma} (y_{\underline{i}} - \overline{y}) \\ \frac{\Sigma}{\Sigma} (x_{\underline{i}} - \overline{x})^{2} \end{bmatrix}^{2} V(x) + \begin{bmatrix} \frac{\Sigma}{\Sigma} (x_{\underline{i}} - \overline{x}) \\ \frac{\Sigma}{\Sigma} (x_{\underline{i}} - \overline{x})^{2} \end{bmatrix}^{2} V(y)$$
(6c)

Therefore:

$$V(b) = \frac{\Sigma(x_{1} - \bar{x})^{2}}{N} \cdot V(m) = \frac{\Sigma(x_{1} - \bar{x})^{2}}{N} \cdot \left\{ \frac{\Sigma(y_{1} - \bar{y})^{2}}{\Sigma(x_{1} - \bar{x})^{2}} \right\} V(x)$$

$$+ \left[\frac{\Sigma(x_{1} - \bar{x})^{2}}{\Sigma(x_{1} - \bar{x})^{2}} \right]^{2} V(y)$$
(6d)

Therefore; in terms of a vs. sin* 4:

$$V(\underline{A}_{1}) = \begin{bmatrix} \frac{\Sigma}{2} (\underline{a}_{1} - \overline{a}_{1}) \\ \frac{\Sigma}{2} (\underline{\sin^{2} \phi_{1}} - \overline{\sin^{2} \phi}) \end{bmatrix}^{2} \qquad V(\underline{\sin^{2} \phi})$$

$$+ \begin{bmatrix} \frac{\Sigma}{2} (\underline{\sin^{2} \phi_{1}} - \overline{\sin^{2} \phi}) \\ \frac{\Sigma}{2} (\underline{\sin^{2} \phi_{1}} - \overline{\sin^{2} \phi}) \end{bmatrix}^{2} \qquad V(\underline{a}_{1}) \qquad (7)$$

The variance in ϕ is negligible, so the first term can be ignored.

Also, from Eq. (2a):

$$V(a_{\underline{a}}) = \begin{bmatrix} \frac{da_{\underline{a}}}{d(d_{\phi\phi+})} \\ a \end{bmatrix}^{2} V(d_{\phi\phi+}) + \begin{bmatrix} \frac{da_{\underline{a}}}{d(d_{\phi\phi-})} \\ d(d_{\phi\phi-}) \end{bmatrix}^{2} V(d_{\phi\phi}) + \begin{bmatrix} \frac{da_{\underline{a}}}{d(d_{\phi})} \\ d(d_{\phi\phi}) \end{bmatrix}^{2} V(d_{\phi\phi})$$
(8)

Writing Bragg's law in the form $d=\frac{\lambda}{2\sin\theta}$, adopting the convention that θ^+ , θ^- are the θ values (in degrees) for the peaks at $+\phi$, $-\phi$ respectively, and employing Eq. (5):

$$V(d_{\phi,\phi^{+}}) = (\pi/180)^{2} (\lambda \cos^{+}/2\sin^{2}\theta^{+})^{2} V(2\theta^{+})/2$$
(9)

and similarly for $V(d_{0.6-})$. Recalling Eq. (2a):

$$\begin{bmatrix} \underline{da} \\ \underline{d(d_{0++})} \end{bmatrix}^2 = \begin{bmatrix} \underline{da_1} \\ \underline{d(d_{0+-})} \end{bmatrix}^2 = \frac{1}{4d_0^2}, \tag{10a}$$

$$-\frac{da_1}{d(d_0)} = \frac{\left[d_{\phi\phi+} + d_{\phi\phi-}\right]^2}{4d_0^4} = d_+^2/4d_0^4. \tag{10b}$$

Thus, we may rewrite Eq. (7):

$$V(\ell_1) = \begin{bmatrix} \frac{\sum_{i=1}^{2} (\sin^2 \phi_i - \frac{1}{\sin^2 \phi_i})}{\sum_{i=1}^{2} (\sin^2 \phi_i - \frac{1}{\sin^2 \phi_i})^2} \end{bmatrix}^2 \frac{1}{4d_0^2} \{ (\pi/180)^2 \frac{\lambda^2}{8} \left[(\frac{\cos \theta^2}{\sin^2 \theta_i})^2 V_i(2\theta^+) + (\frac{\cos \theta^-}{\sin^2 \theta_i})^2 V_i(2\theta^-) \right] + (d_+^2/d_0^2) V(d_0) \}$$

$$(11)$$

In a similar manner for a_2 vs. $\sin |2\psi|$, where $a_2 \equiv (d_{\phi\psi_-} - d_{\phi\psi_-})/2d_0 = d^-/2d_0$:

$$V(\ell_{2}) = \begin{bmatrix} \frac{\sum \sin|2\phi_{1}| - \frac{1}{\sin|2\phi|}}{\sum (\sin|2\phi_{1}| - \sin|2\phi|)^{2}} \end{bmatrix}^{2} \frac{1}{\frac{1}{4d_{0}^{2}}} \{ (\frac{\pi}{180})^{2} (\frac{\lambda^{2}}{8}) [(\frac{\cos\theta^{+}}{\sin^{2}\theta^{+}})^{2} V_{1}(2\theta^{+}) + (\frac{\cos\theta^{-}}{\sin^{2}\theta^{-}})^{2} V_{1}(2\theta^{-})] + (\frac{d^{2}}{4d_{0}^{2}}) V(d_{0}) \}$$

$$= \frac{\left[\sum \sin|2\phi_{1}| - \frac{\sin|2\phi_{1}|}{\sin^{2}\theta^{-}} + \frac{1}{4d_{0}^{2}} (\frac{\pi}{180})^{2} (\frac{\lambda^{2}}{8}) [(\frac{\cos\theta^{+}}{8})^{2} V_{1}(2\theta^{+}) + (\frac{\cos\theta^{-}}{8})^{2} V_{1}(2\theta^{-})] + (\frac{d^{2}}{4d_{0}^{2}}) V(d_{0}) \}$$

$$= \frac{\left[\sum \sin|2\phi_{1}| - \frac{\sin|2\phi_{1}|}{\sin^{2}\theta^{+}} + \frac{1}{4d_{0}^{2}} (\frac{\pi}{180})^{2} (\frac{\lambda^{2}}{8}) [(\frac{\cos\theta^{+}}{8})^{2} V_{1}(2\theta^{+}) + (\frac{\cos\theta^{-}}{8})^{2} V_{1}(2\theta^{-})] + (\frac{d^{2}}{4d_{0}^{2}}) V(d_{0}) \}$$

$$= \frac{\left[\sum \sin|2\phi_{1}| - \frac{\sin|2\phi_{1}|}{\sin^{2}\theta^{+}} + \frac{1}{4d_{0}^{2}} (\frac{\pi}{180})^{2} (\frac{\lambda^{2}}{8}) [(\frac{\cos\theta^{+}}{8})^{2} V_{1}(2\theta^{+}) + (\frac{\cos\theta^{-}}{8})^{2} V_{1}(2\theta^{-})] + (\frac{d^{2}}{4d_{0}^{2}}) V(d_{0}) \}$$

$$= \frac{1}{4d_{0}^{2}} \left[\sum \left(\sum \frac{\cos\theta^{+}}{8} + \frac{\cos\theta^{-}}{8} + \frac{\cos\theta^{-}}{8}$$

We now propagate these values into the strain and stress tensors.

THE STRAIN TENSOR

Abbreviating the intercept of a_i vs. $\sin^2 \phi$ as I, then at any Φ :

$$\varepsilon_{33} = I \left(\text{of } \mathbf{s}_1 \text{ vs. } \sin^2 \phi \right), \tag{13a}$$

$$V(\varepsilon_{3,3}) = V(\ell_1) + V(I)$$
 (13b)

$$V(I) = \frac{\sum (\sin^2 \phi_i - \overline{\sin^2 \phi})^2}{N} \cdot \left[\frac{\sum (\sin^2 \phi_i - \overline{\sin^2 \phi})}{\sum (\sin^2 \phi_i - \overline{\sin^2 \phi})^2} \right]^2 \quad V(a_{i})$$

$$= \frac{1}{N} \cdot \frac{\left[\sum (\sin^2 \phi_i - \overline{\sin^2 \phi})\right]^2}{\sum (\sin^2 \phi_i - \overline{\sin^2 \phi})^2} \quad V(a_{i}) \quad (13c)$$

Now, from Eqs. (3c), at $\Phi = 0^{\circ}$:

$$e^{\ell_1} = e_{11} - e_{33} = e_{11} - I, \qquad (14)$$

$$V(\epsilon_{11}) = 2V(e^{\ell_1}) + V(I).$$

Similarly, for ● = 90°:

$$V(e_{22}) = V(_{90} l_1) + V(_{0} l_2) + V(I),$$
 (15)

and for $\bullet = 45^{\circ}$:

$$\varepsilon_{12} = {}_{45} \mathcal{L}_{1} + \varepsilon_{23} - 0.5 (\varepsilon_{11} + \varepsilon_{22})
+ {}_{45} {}_{1} - 0.5 ({}_{6} \mathcal{L}_{1} + {}_{96} \mathcal{L}_{1}),
V(\varepsilon_{12}) = V({}_{45} \mathcal{L}_{1}) + 0.25 [V({}_{9} \mathcal{L}_{1}) + V({}_{96} \mathcal{L}_{1})].$$
(16)

From Eqs. (3d):

$$V(e_{12}) = V(q e_2), \qquad (17)$$

$$V(e_{23}) = V(e_{24}). \tag{18}$$

THE STRESS TENSOR

We define $Q = S_1/(3S_1 + 1/2S_2)$ (which is [" $^{-V}/1-2v$] for an isotropic solid). Then Eq. (4) may be written, for the diagonal stress components, as:

$$\sigma_{ij} = (1/2S_z)^{-1}[(1-Q)e_{ii} - Qe_{kk} - Qe_{jj}].$$
 (19)

Here i = 1,2,3; j = 2,3,1; k = 3,1,2.

From Eq. (19):

$$V(\sigma_{ii})^{\frac{1}{2}} = (1/2s_2)^{-1} \{(1-Q)^2 V(\epsilon_{ii}) + Q^2 [V(\epsilon_{kk}) + V(\epsilon_{jj})]\}^{\frac{1}{2}}.$$
 (20)

Therefore, with Eqs. (13-15):

$$V(\sigma_{11})^{\frac{1}{2}} = (1/2S_2)^{-1} \{(2-4Q + 4Q^2)V(_0 \ell_1) + Q^2V(_{90} \ell_1) + (1-2Q + 3Q^2 V(1))^{\frac{1}{2}},$$
 (21)

$$V(\sigma_{22})^{\frac{1}{2}} = (1/2S_2)^{-1} \left\{ (1-2Q + 4Q^2)V(_{0} \ell_{1}) + (1-2Q + Q^2)V(_{9} e \ell_{1}) + (1-2Q + 3Q^2)V(1) \right\}^{\frac{1}{2}},$$
(22)

$$V(\sigma_{3,3})^{\frac{1}{2}} = (1/2S_2)^{-1} \{ (1-2Q + 4Q^2)V(_0 L_1) + Q^2V(_{3,0} L_1) + (1-2Q + 3Q^2)V(1) \}^{\frac{1}{2}}$$
(23)

Similarly:

$$V(\sigma_{12})^{\frac{1}{2}} = (1/2S_2)^{-1} [V(s_1, l_1) + 0.25 [V(s_1, l_1) + V(s_1, l_1)]^{\frac{1}{2}},$$
 (24)

$$V(\sigma_{13})^{\frac{1}{2}} = (1/2S_2)^{-1}V(\rho_{23})^{\frac{1}{2}}, \qquad (25)$$

$$V(\sigma_{23}) = (1-2S_2)^{-1}V(s_0 \ell_2)^{\frac{1}{2}}.$$
 (26)

EXAMPLES

To illustrate the typical magnitudes of the errors due to counting statistics, we employed data from Ref. 5, for a ground steel specimen, that is we used the peak positions and the variances in these positions with Eq. (9). [Formulae to calculate this variance for the parabolic fit employed in Ref. 5 are given in Ref. 1; for other types of fits the appropriate equation may be substituted.] The resultant errors are given in Tables I-III. For the first two tables it was assumed that the error in $\mathbf{d}_{\phi, \phi}$ was the actual measured value. If there is no preferred orientation, the intensity of the peak changes little with the ϕ -tilt. In this case, Tables I and II show the effect of the uncertainty in \mathbf{d}_{ϕ} ; reducing this error all the stress components by the same proportion, except σ_{13} , σ_{23} , which remain relatively unaffected, because the role of the error in \mathbf{d}_{ϕ} is damped by $(\mathbf{d}_{\phi})^2$ in Eq. (12).

If there is preferred orientation, the peak intensity can vary greatly with ϕ and there will be large variances contributing to $V(\ell_1)$ from weak peaks. This was minimized in the following way. The average variance, σ_1 , in the 20 peak position for $+\phi$ and $-\phi$ was obtained and the weighting factor c_1 was formed:

$$c_i = (1/\sigma_i^2) / \sum_i (1/\sigma_i^2)$$
 (27)

The Eqs. 11 and 12 were then altered to multiply $V_i(2\theta^+)$, $V_i(2\theta^-)$ terms by this weighting for Table III. There is only a small difference (between Tables II and III) because of the lack of texture in the specimen; the peak intensity changed only by about 8 pct with ϕ . With more severe preferred orientation the effect will be larger.

CONCLUDING REMARKS

There are now adequate equations for calculating errors in stress

TABLE II: STRESS TENSOR AND STANDARD DEVIATIONS WHEN $V(d_0)^{\frac{1}{2}} = 0.00004 \text{ Å}^{\frac{1}{2}}$

		DATA	SET 1		
539.74	(48.24)	-24.03	(24.50)	-39.15	(4.58)
		552.16	(47.26)	2.30	(3.56)
				80.41	(38.96)
		DATA	SET 2		
520.60	(40.52)	-4.03	(19.95)	-34.17	(3.21)
		555.19	(39.73)	0.11	(2.69)
				82.20	(32.75)
		DATA	SET 3		
535.03	(47.81)	-20.14	(24.35)	-40.19	(5.72)
		555.98	(46.81)	-0.98	(4.56)
				86.66	(38.84)
		DATA	SET 4		
538.53	(42.84)	-30.63	(21.10)	-38.03	(3.83)
		565.37	(42.51)	0.76	(3.89)
				88.18	(34.67)

*values given in MPa

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TABLE III: WEIGHTED STRESS TENSOR AND STANDARD DEVIATIONS*

		DATA	SET 1		
536.24	(48.56)	-24.62	(24.56)	-38.33	(4.59)
		554.65	(47.60)	2.90	(3.65)
			• • • • • • • • • • • • • • • • • • • •	80.68	(39.22)
		DATA :	SET 2		
520.29	(41.84)		(20.11)	-34.82	(3.55)
			(40.60)	1.56	(2.73)
		00000	(14400)	85.29	(33.76)
		DATA :	SET 3		
532.56	(48.15)	-7.90	(24.93)	-39.66	(5.81)
		549.83	(47.16)	-2.81	(4.57)
			• • • • • • • • • • • • • • • • • • • •	82.21	(39.16)
		DATA	SET 4		
539.23	(42.88)	-31.22	(21.23)	-38.28	(3.85)
	•	565.17		-0.59	(3.94)
			, , , , , , , , , , , , , , , , , , , ,	88.98	(34.68)

^{*} $V(d_0)^{\frac{1}{2}} = 0.00004 \text{ Å}$; values given in MPa

measurements due to instrumental effects, counting statistics and in the x-ray elastic constants. We would like to conclude this paper with a plea to the community making stress measurements via diffraction to regularly report these errors with their data. It is all too common for investigators to repeat a measurement (of stress or an elastic constant) once and to use the difference as an error estimate. Another practice is to dust a stress-free powder on the specimen surface and to use a (single) measurement of the stresses measured with this powder as an error estimate. Finally, some report an error in a slope vs. $\sin^2 \phi$ obtained by least-squares, but ignore the uncertainty in each point in this plot in estimating errors. None of these procedures is particularly satisfying in a statistical sense. Of course, if time permits, the average of, say, ten repetitions of a measurement is the best of all error estimates. If this cannot be done, error estimates from the available equations are far more satisfactory than the currently all - too common procedures.

ACKNOWLEDGEMENTS

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REFERENCES

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- 1) M. R. James and J. B. Cohen, Study of the Precision of X-ray Stress Analysis, Adv. X-ray Analysis, 20:291(1977).
- 2) R. H. Marion, "X-ray Stress Analysis in Plastically Deformed Metals", Ph.D. Thesis, Northwestern University (1973).
- 3) S. Taira, T. Abe and T. Ehiro, X-ray Study of Surface Residual Stress Produced in Fatigue Process of Annealed Metals, <u>Bull J.S.M.E.</u>, 12:53, 947(1969).
- 4) K. Perry, I. C. Noyan, P. J. Rudnik and J. B. Cohen, "The Measurement of Elastic Constants for the Determination of Stresses by X-rays, Adv. X-ray Analysis, 27: 159(1984).
- 5) H. Dolle and J. B. Cohen, Residual Stresses in Ground Steels, Met. Trans., 11A:831(1980).
- 6) I. C. Noyan, Equilibrium Conditions for the Average Stress Measured by X-rays, Met. Trans., 14A:1907(1983).
- 7) A. D. Krawitz, J. E. Brune and M. J. Schmank, Measurement of Stress in the Interior or of Solids with Neutrons in: "Residual Stresses and Stress Relaxation", E. Kula and V. Weiss eds., Plenum Press New York (1982).
- 8) I. C. Noyan, Determination of the Unstressed Lattice Parameter, "a," for (Triaxial) Residual Stress Determined by X-rays, Adv. X-ray Analysis, 28: (1985).
- 9) O. Davies and P. Goldsmith, Statistical Methods in Research and Production, Hafner Publ. Co., New York (1952).

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